Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced
Further Mathematics Option 1
Paper 3: Decision Mathematics 1
Further Mathematics Option 2
Paper 4: Decision Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference
9FM0/3D
9FM0/4D

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the answer book provided.

1. A list of n numbers needs to be sorted into descending order starting at the left-hand end of the list.

(a) Describe how to carry out the first pass of a bubble sort on the numbers in the list.

(2)

Bubble sort is a quadratic order algorithm.

A computer takes approximately 0.021 seconds to apply a bubble sort to a list of 2000 numbers.

(b) Estimate the time it would take the computer to apply a bubble sort to a list of 50 000 numbers. Make your method clear.

(2)

(Total for Question 1 is 4 marks)

1.

a) 1. compare 1st value with 2nd value in list

2. if 2nd value is larger than 1st value, swap items

3. compare value now in 2nd with 3rd value in list

4. swap if the 3rd is larger than the 2nd

5. continue in this way until end of list

b) quadratic so time $t \propto n^2$

therefore

$$t = 0.021 \times \left(\frac{50000}{2000}\right)^2$$

$$\underline{t = 13.125\text{ s}}$$

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Question 1 continued

(Total for Question 1 is 4 marks)

2.

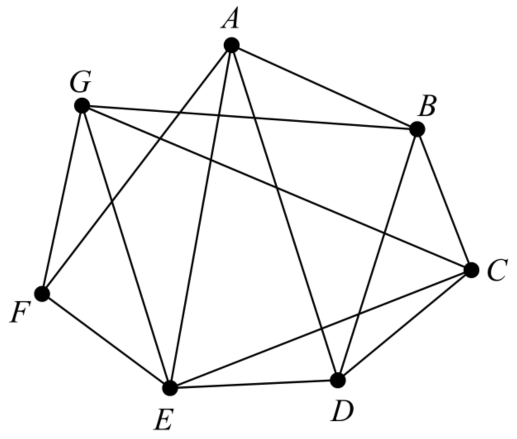

Figure 1

(a) Define what is meant by a **planar** graph.

(2)

(b) Starting at A, find a Hamiltonian cycle for the graph in Figure 1.

(1)

Arc AG is added to Figure 1 to create the graph shown in Figure 2.

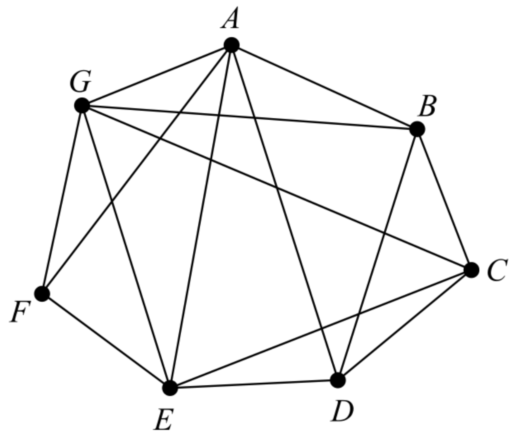

Figure 2

Taking ABCDEFGA as the Hamiltonian cycle,

(c) use the planarity algorithm to determine whether the graph shown in Figure 2 is planar. You must make your working clear and justify your answer.

(4)

(Total for Question 2 is 7 marks)

2.

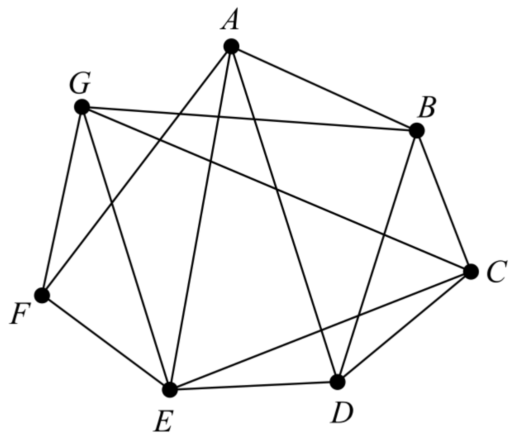

Figure 1

a) a planar graph is a graph that can be drawn such that no arc meets another arc except at a vertex

b) e.g. ABCDEGFA

(must contain 8 vertices, with every vertex except A appearing only once)

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Question 2 continued

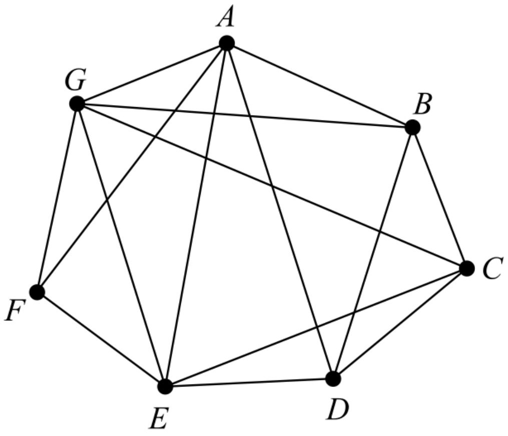
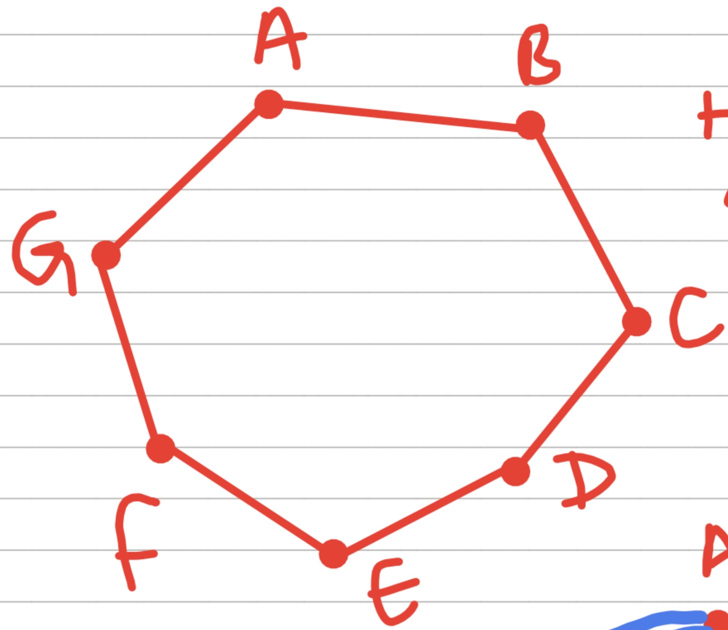


Figure 2

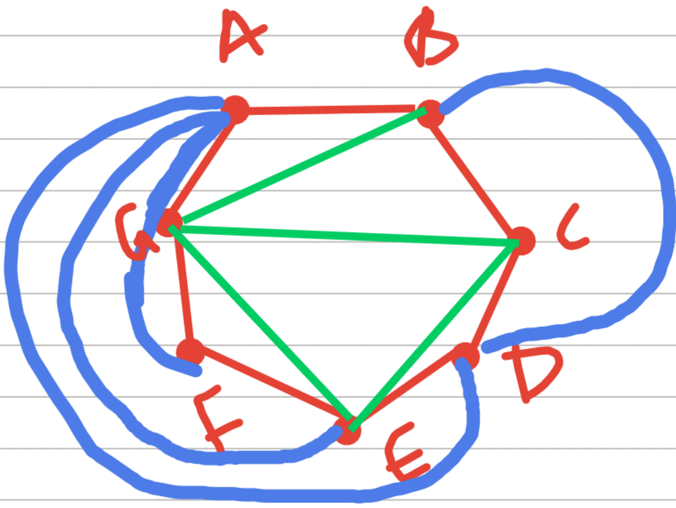
c)



Hamiltonian cycle

remaining arcs:

in:	out:
BG	AD
CG	BD
EG	AE
CE	AF



no arcs in list cross & no arc appears in both lists so graph is planar

(Total for Question 2 is 7 marks)

3. (a) Explain clearly the difference between the classical travelling salesperson problem and the practical travelling salesperson problem.

(2)

	A	B	C	D	E	F	G
A	–	17	24	16	21	18	41
B	17	–	35	25	30	31	x
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

The table shows the least distances, in km, by road between seven towns, A, B, C, D, E, F and G. The least distance between B and G is x km, where $x > 25$

Preety needs to visit each town at least once, starting and finishing at A. She wishes to minimise the total distance she travels.

(b) Starting by deleting B and all of its arcs, find a lower bound for Preety's route.

(3)

Preety found the nearest neighbour routes from each of A and C. Given that the sum of the lengths of these routes is 331 km,

(c) find x, making your method clear.

(4)

(d) Write down the smallest interval that you can be confident contains the optimal length of Preety's route. Give your answer as an inequality.

(2)

(Total for Question 3 is 11 marks)

3. a) practical: each vertex visited at least once

classical: " " just once

	A	B	C	D	E	F	G
	1		5	2	4	3	6
A	–	17	24	16	21	18	41
~~B~~	~~17~~	~~–~~	~~35~~	~~25~~	~~30~~	~~31~~	~~x~~
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

b) Prim's algorithm

1. AD 2. AF 3. AE 4. CE 5. CG

add B back in using AB & BD

lower bound = tree + AB + BD

= 107 + 17 + 25

= 149 km

Question 3 continued

	A	B	C	D	E	F	G
A	–	17	24	16	21	18	41
B	17	–	35	25	30	31	x
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

c) nearest neighbour A:

ADFECGBA

from C:

CEADFBGC

from A $= 126 + x$

from C $= 139 + x$

so $126 + x + 139 + x = 331 \Rightarrow \underline{x = 33}$

d) $149 < \text{optimal} \leq 159$

$126 + 33 = 159$

from part c

(Total for Question 3 is 11 marks)

4.

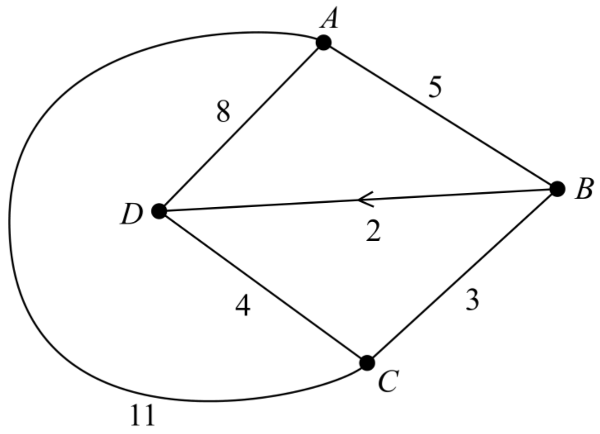


Figure 3

The network in Figure 3 shows the roads linking a depot, D, and three collection points A, B and C. The number on each arc represents the length, in miles, of the corresponding road. The road from B to D is a one-way road, as indicated by the arrow.

(a) Explain clearly if Dijkstra's algorithm can be used to find a route from D to A.

(1)

The initial distance and route tables for the network are given in the answer book.

(b) Use Floyd's algorithm to find a table of least distances. You should show both the distance table and the route table after each iteration.

(7)

(c) Explain how the final route table can be used to find the shortest route from D to B. State this route.

(2)

There are items to collect at A, B and C. A van will leave D to make these collections in any order and then return to D. A minimum route is required.

Using the final distance table and the Nearest Neighbour algorithm starting at D,

(d) find a minimum route and state its length.

(2)

Floyd's algorithm and Dijkstra's algorithm are applied to a network. Each will find the shortest distance between vertices of the network.

(e) Describe how the results of these algorithms differ.

(2)

(Total for Question 4 is 14 marks)

4.

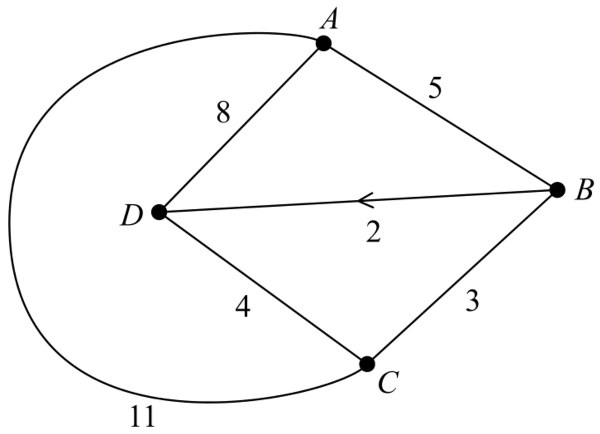


Figure 3

a) yes, Dijkstra's algorithm can be applied to either a directed or undirected network

(b)

	A	B	C	D
A	–	5	11	8
B	5	–	3	2
C	11	3	–	4
D	8	∞	4	–

	A	B	C	D
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	B	C	D

	A	B	C	D
A	–	5	11	8
B	5	–	3	2
C	11	3	–	4
D	8	13	4	–

	A	B	C	D
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	A	C	D

	A	B	C	D
A	–	5	8	7
B	5	–	3	2
C	8	3	–	4
D	8	13	4	–

	A	B	C	D
A	A	B	B	B
B	A	B	C	D
C	B	B	C	D
D	A	A	C	D

Question 4 continued

	A	B	C	D
A	–	5	8	7
B	5	–	3	2
C	8	3	–	4
D	8	7	4	–

	A	B	C	D
A	A	B	B	B
B	A	B	C	D
C	B	B	C	D
D	A	C	C	D

	A	B	C	D
A	–	5	8	7
B	5	–	3	2
C	8	3	–	4
D	8	7	4	–

	A	B	C	D
A	A	B	B	B
B	A	B	C	D
C	B	B	C	D
D	A	C	C	D

no changes → optimal

c) start in row D & read across to column B

box reads C → route begins DC

go to row C, column B → reads B

so route is DCB

d) using nearest neighbour

D – C – B – A – B – D

$4 + 3 + 5 + 5 + 2 = 19$ miles

(Total for Question 4 is 14 marks)

5. A garden centre makes hanging baskets to sell to its customers. Three types of hanging basket are made, *Sunshine*, *Drama* and *Peaceful*. The plants used are categorised as *Impact*, *Flowering* or *Trailing*.

Each *Sunshine* basket contains 2 *Impact* plants, 4 *Flowering* plants and 3 *Trailing* plants.
Each *Drama* basket contains 3 *Impact* plants, 2 *Flowering* plants and 4 *Trailing* plants.
Each *Peaceful* basket contains 1 *Impact* plant, 3 *Flowering* plants and 2 *Trailing* plants.

The garden centre can use at most 80 *Impact* plants, at most 140 *Flowering* plants and at most 96 *Trailing* plants each day.

The profit on *Sunshine*, *Drama* and *Peaceful* baskets are £12, £20 and £16 respectively. The garden centre wishes to maximise its profit.

Let x, y and z be the number of *Sunshine*, *Drama* and *Peaceful* baskets respectively, produced each day.

(a) Formulate this situation as a linear programming problem, giving your constraints as inequalities.

(5)

(b) State the further restriction that applies to the values of x, y and z in this context.

(1)

The Simplex algorithm is used to solve this problem. After one iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$-\frac{1}{4}$	0	$-\frac{1}{2}$	1	0	$-\frac{3}{4}$	8
s	$\frac{5}{2}$	0	2	0	1	$-\frac{1}{2}$	92
y	$\frac{3}{4}$	1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	24
P	3	0	–6	0	0	5	480

(c) State the variable that was increased in the first iteration. Justify your answer.

(2)

(d) Determine how many plants in total are being used after only one iteration of the Simplex algorithm.

(1)

(e) Explain why for a second iteration of the Simplex algorithm the 2 in the z column is the pivot value.

(2)

After a second iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$\frac{3}{8}$	0	0	1	$\frac{1}{4}$	$-\frac{7}{8}$	31
[redacted]	$\frac{5}{4}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	46
y	$\frac{1}{8}$	1	0	0	$-\frac{1}{4}$	$\frac{3}{8}$	1
P	$\frac{21}{2}$	0	0	0	3	$\frac{7}{2}$	756

(f) Use algebra to explain why this tableau is optimal.

(1)

(g) State the optimal number of each type of basket that should be made.

(1)

The manager of the garden centre is able to increase the number of *Impact* plants available each day from 80 to 100. She wants to know if this would increase her profit.

(h) Use your final tableau to determine the effect of this increase. (You should not carry out any further calculations.)

(2)

(Total for Question 5 is 15 marks)

5. a) maximise $P = 12x + 20y + 16z$

Subject to $2x + 3y + z \leq 80$

$4x + 2y + 3z \leq 140$

$3x + 4y + 2z \leq 96$

$x, y, z \geqslant 0$

b) integer values required

c) y was increased first, because it is now in the basic variable column

d) $r = 8$, $s = 92$, $t = 0$

$r + s + t$

so total $= \underbrace{80 + 140 + 96}_{} - \underbrace{(8 + 92)}_{}$
$= 216$ plants

maximum possible values

e) we choose the pivot column to be the column with the most negative value in the objective row, so it must be column z

pivot must be positive & the lowest of $92 \div 2 = 46$ & $24 \div 0.5 = 48$, so pivot is the 2 from column z

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Question 5 continued

f) $P + 10.5x + 3s + 3.5t = 756$

$\Rightarrow$ increasing x, s, or t decreases profit P

g) 1 Drama

46 Peaceful

h) the associated slack variable is r, & has a value of 31.

$20 < 31$, so increasing x by 20 would have no effect

(Total for Question 5 is 15 marks)

6.

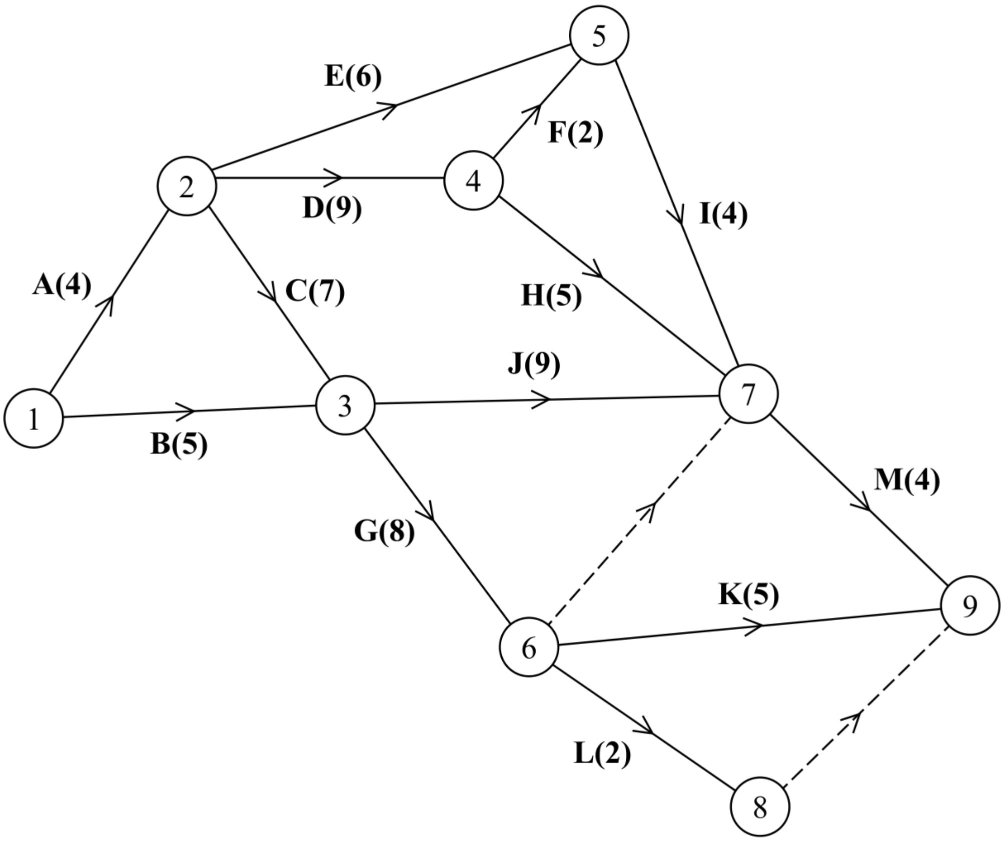


Figure 4

A project is modelled by the activity network shown in Figure 4. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete that activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Calculate the early time and the late time for each event, using Diagram 1 in the answer book.

(3)

(b) On Grid 1 in the answer book, complete the cascade (Gantt) chart for this project.

(3)

(c) On Grid 2 in the answer book, draw a resource histogram to show the number of workers required each day when each activity begins at its earliest time.

(3)

The supervisor of the project states that only three workers are required to complete the project in the minimum time.

(d) Use Grid 2 to determine if the project can be completed in the minimum time by only three workers. Give reasons for your answer.

(3)

(Total for Question 6 is 12 marks)

6.

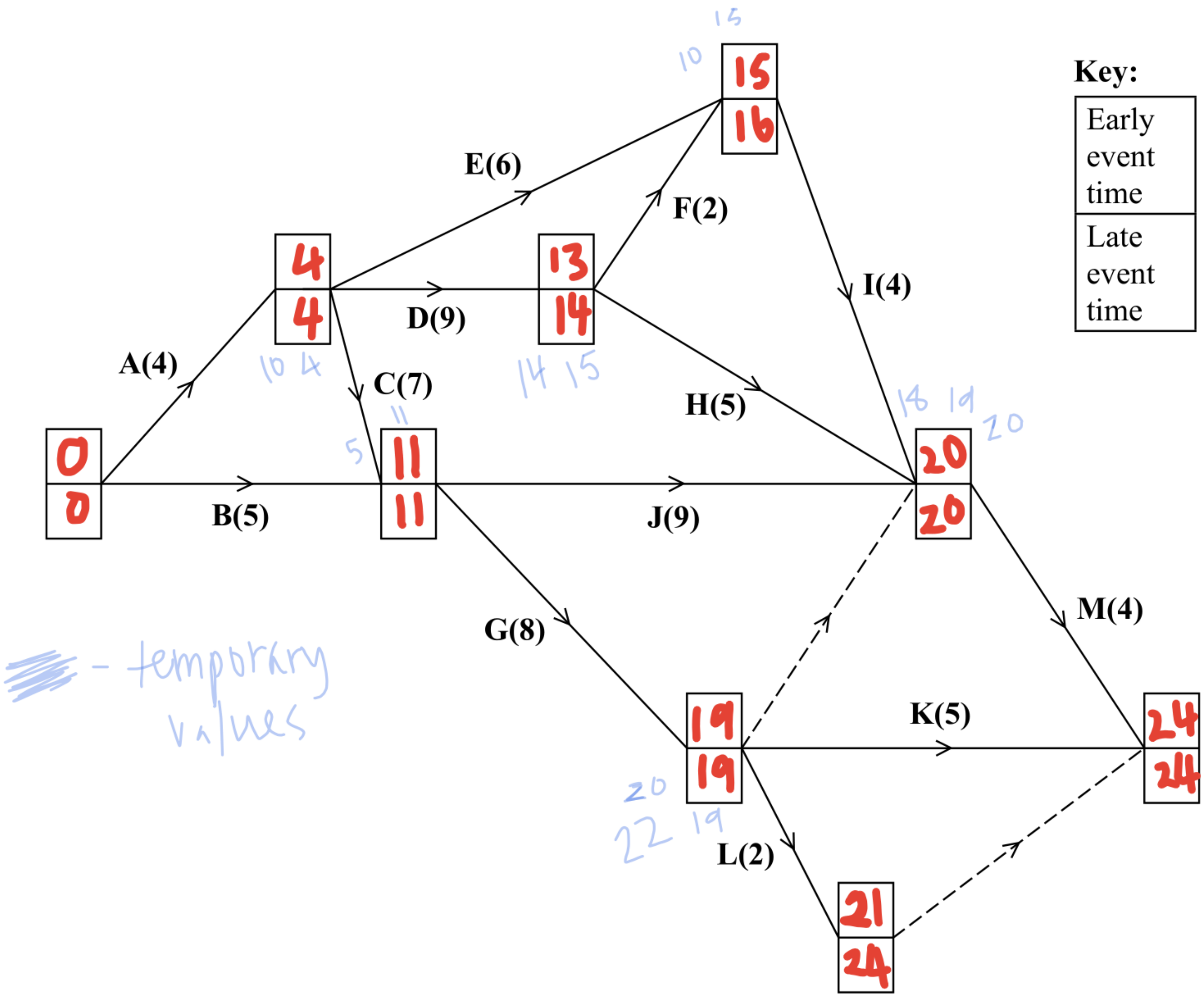
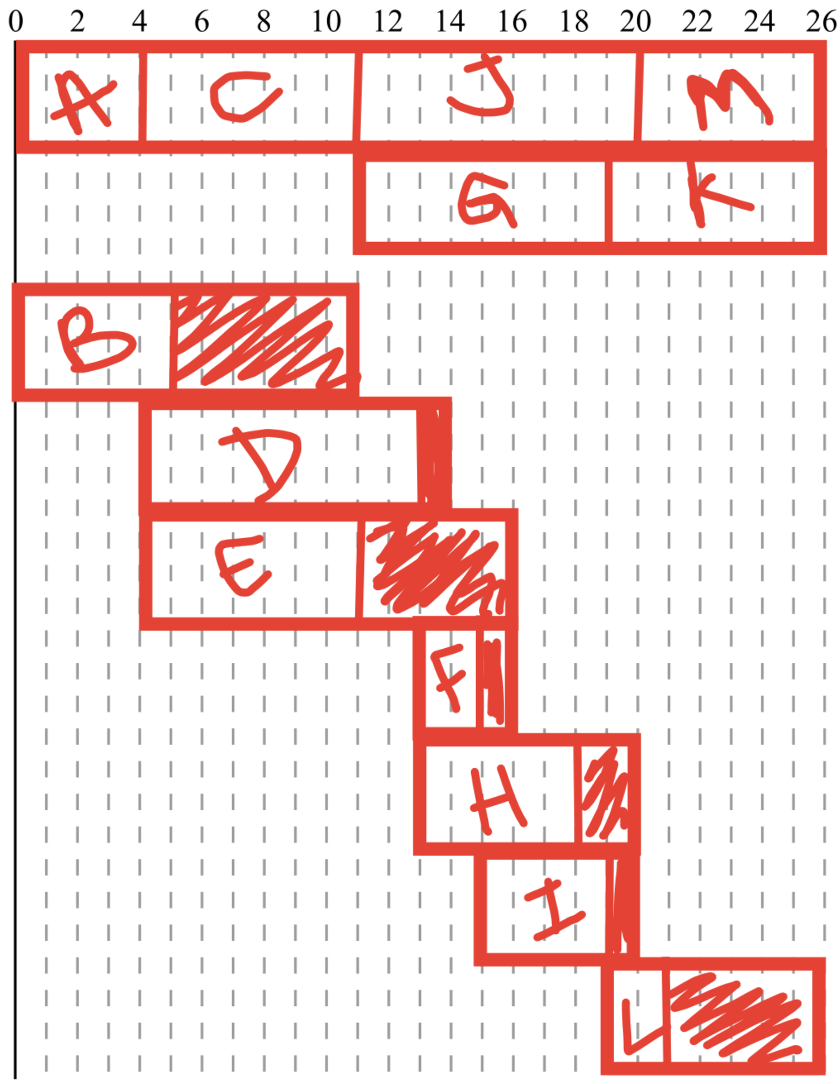


Diagram 1

b) critical paths: A-C-JM, A-C-GK

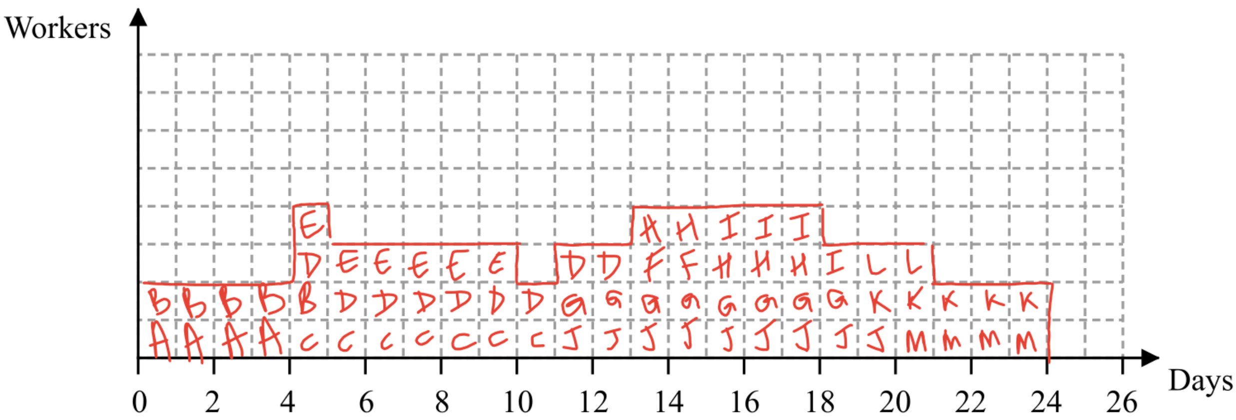
c) until time 4, only A & B can happen, so the spare worker can't be used. after time 4, there are times when 4 events must be happening for there to be no delays ⇒ project cannot be finished by time 24 with 3 workers

Question 6 continued



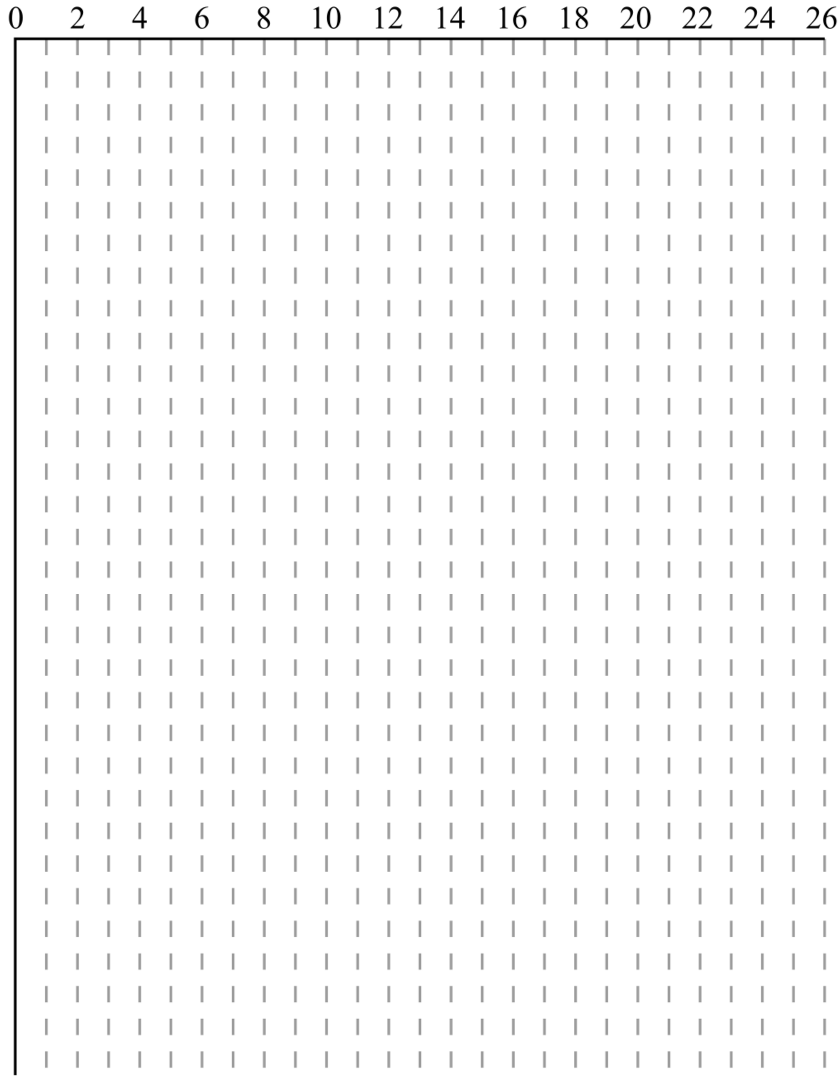
Grid 1

(There is a spare grid on the next page)



Grid 2

Question 6 continued



Copy of Grid 1

(Total for Question 6 is 12 marks)

7. A linear programming problem in x, y and z is described as follows.

Maximise $P = 3x + 2y + 2z$

subject to $\quad 2x + 2y + z \leqslant 25$

$x + 4y \leqslant 15$

$x \geqslant 3$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

The big-M method is to be used to solve this linear programming problem.

(b) Define, in this context, what M represents. You must use correct mathematical language in your answer.

(1)

The initial tableau for a big-M solution to the problem is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	2	2	1	1	0	0	0	25
s_2	1	4	0	0	1	0	0	15
t_1	1	0	0	0	0	-1	1	3
P	$-(3 + M)$	-2	-2	0	0	M	0	$-3M$

(c) Explain clearly how the equation represented in the b.v. t_1 row was derived.

(1)

(d) Show how the equation represented in the b.v. P row was derived.

(2)

The tableau obtained from the first iteration of the big-M method is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	0	2	1	1	0	2	–2	19
s_2	0	4	0	0	1	1	–1	12
x	1	0	0	0	0	–1	1	3
P	0	–2	–2	0	0	–3	$3 + M$	9

(e) Solve the linear programming problem, starting from this second tableau. You must

- give a detailed explanation of your method by clearly stating the row operations you use and
- state the solution by deducing the final values of P, x, y and z.

(7)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

a) the problem contains a $\geq$ constraint, but simplex only works for $\leq$ constraints

b) M = arbitrarily large, real number

c) $x \geq 3 \rightarrow x - s_3 + t_1 = 3$

s_3: surplus variable t_1: artificial variable

d) if $P = 3x + 2y + 2z - Mt_1 \Rightarrow$

$P = 3x + 2y + 2z - M(3 - x + s_3)$

$= (3+M)x + 2y + 2z - Ms_3 - 3M$

$\rightarrow P - (3+M)x - 2y - 2z + Ms_3 = -3M$

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	0	2	1	1	0	2	−2	19
s_2	0	4	0	0	1	1	−1	12
x	1	0	0	0	0	−1	1	3
P	0	−2	−2	0	0	−3	$3 + M$	9

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
s_3	0	1	1/2	1/2	0	1	−1	19/2	$R_1 \times \frac{1}{2}$
s_2	0	3	−1/2	−1/2	1	0	0	5/2	$R_2 - R_1$
x	1	1	1/2	1/2	0	0	0	25/2	$R_3 + R_1$
P	0	1	−1/2	3/2	0	0	M	75/2	$R_4 + 3R_1$

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
z	0	2	1	1	0	2	−2	19	$2R_1$
s_2	0	4	0	0	1	1	−1	12	$R_2 + \frac{1}{2}R_1$
x	1	0	0	0	0	−1	1	3	$R_3 - \frac{1}{2}R_1$
P	0	2	0	2	0	1	$M - 1$	47	$R_4 + \frac{1}{2}R_1$

Question 7 continued

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
P									

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
P									

e) all objective values $\geq 0 \Rightarrow$

$P = 47$

$x = 3$

$y = 0$

$z = 19$

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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